Designing a unit of study to develop mathematical critical thinking – "What does it mean to be average?"

The proposed design of this unit on average pulls together some of the disparate experiences and learnings of the action research teachers to create a coherent whole. Teachers explain in italics how they ran the activities and why, and what they found.

1. Guiding question "What is average?"

2. Finding out what students know What are some of the ways that students think about averages both mathematically and as part of everyday language?

"I asked students to list 5 things they knew about average in pairs. Some students could only think of one or two, while other students came up with 5 answers. I was quite surprised about the range of meanings. However now having seen the research on student preconceptions I think my students' responses were fairly typical."

What does the term "Average" mean to you? Write down 5 dot points to help you explain.

- To sum everything up
- An Average is what people Should be at or above
- Averages to give you an idea in what you should be around. Eg. weight, Height, etc.
- An Average is what is in between Such as in the middle of good & bad
- Averages can be displayed in percentage

3. Introducing terminology *(De-coding)* Following the exploration of students' intuitions it is important to provide definitions of different averages - mean, mode and median. This includes exploring the advantages and disadvantages of using each one.

"After introducing the three definitions of Mean, Mode and Median with students I conducted this activity. Students were each assigned an annual income, ranging from single parent pensioner to billionaire. Through discussion we tried to create a 'realistic' cross-section of society, so there was one billionaire, several executives and successful businesspeople, a range of middle income workers and a variety of low income groups.

The students lined up from poorest to richest and found most of the money was held by a relatively small group at the 'wealthy end'. We discussed who would best be described as an 'average' person, the students locating it around the

middle of the line. However when the mean income was calculated it was found to be much closer to the 'wealthy end' of the line.

The students were able to explain why the actual mean was located away from their predicted location, and concluded that Mean was not the best descriptor in this case. Some felt Mode would better describe the 'average' person's income, while others preferred to use Median. On investigation, Mode was found in this example to have a slightly higher income than Median.

This led to a discussion of whether this was a 'fair test', and how data could be collected to make it so. The conclusion was that actual figures for a much larger sample would be needed. They were able to suggest several methods of collection including surveys, and accessing ABS figures.

I found this a very useful exercise as it was an example to which students could relate, and which allowed them to visualise and apply the concepts of Summary Statistics."

4. Meet the Averages *(De-coding and Meaning-making)* Using the article *'Meet the averages'* from The Mercury Newspaper (Stevenson, 1998) the following questions can be asked of students. Which of these statements do you think comes from mean, mode or median? Why do you think they might be used? How useful is this data? What are you thinking about as you read it? Who do you think might use this?
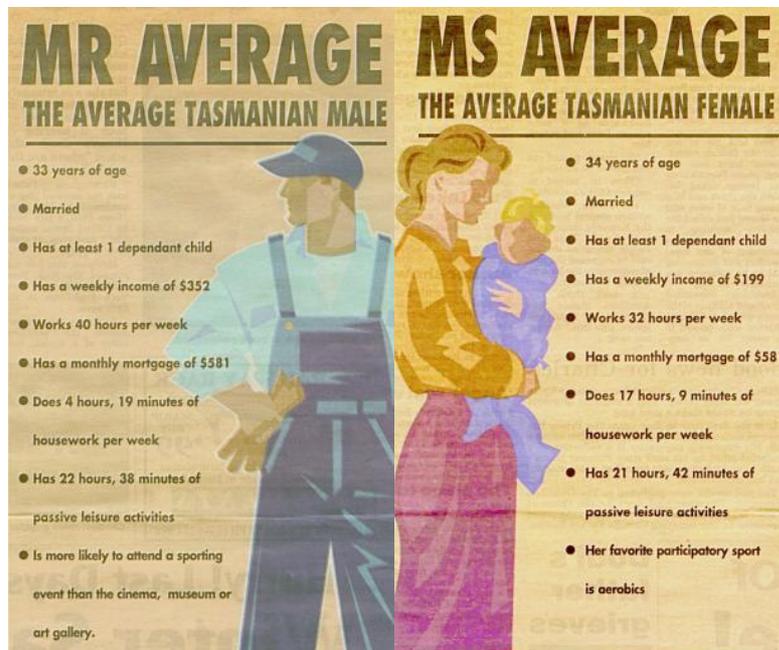

"This approach helped students to develop some criteria for working out which average is being used. The ones with explicit numbers were more likely to be mean, and those that were categories were more likely to be mode. We had problems though wondering whether median was used instead of mean."

5. What's average in my class?
(*Meaning-making and Using*) After the initial de-coding and meaning-making discussion students now consider some questions in context of their own experience. Following on from examining the news article on Mr and Mrs Average what questions would you ask to find out what is average in your class? Collect data to construct a picture describing who is the average student in your class and write a newspaper article.

"I found this generated very useful discussion on what makes a good question. For example some students were interested in finding out what cars students had in their families and wanted other students to give the make and model of the cars. We soon realised that this was far too much detail and we needed to categorise it more broadly into just the manufacturer.
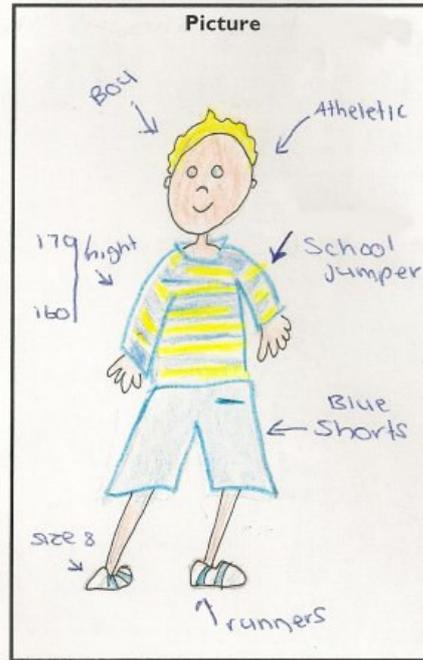
We ended up putting our possible questions on the board and critiquing them before each student decided on their own question. The whole class discussion was an important aspect of this exercise. We then used the board to collect all the data from the class so all students could use it. They then had to decide in pairs on whether to analyse the data using mean, mode or median. Based on that each pair drew their own average classmate and wrote a newspaper article based on what the average student was.

One group found the mean shoe size to be 7.86 and thought this was very funny. They used this number in their article to be deliberately ironical, showing, I felt, a connected sense of what the numbers actually mean.

As an extension for some of my brighter students I asked them could they choose whether to use the mean, mode or median in such a way to make the average class student to be more like them? They took up the challenge, playing around with the numbers. I have begun to recognise how important it is for students to have time to be playful with the numbers and really get inside them."

6. Encouraging critical questioning (*Using and Analysing*) Once students have experienced using the different forms of average they can bring a more critical lens to their use in the media?. The following illustrate possible approaches:

- **Whole class discussion** The following questions could be starters for a discussion. How useful is it to work out the average of a group of people? Is it useful to have an average as 7.86 foot-size? Who might use these summaries? For example, could the canteen use the information on the

favourite food for the class? What is missing that would make the information more useful – e.g., graphs showing variation and more details. How might the questions about an average person change over time? (What questions might be asked in 1960, 2010 and 2060?) What are the issues in comparing yourself with an average?

- **Pair analysis** If pairs of students in the class have written articles about on the "average classmate" these can be shared. Taking another pair's article critiques can be written asking: What is misleading, missing, useful? How are the writers trying to position us? These can be presented to the class for further discussion.

- **Meta-cognition** In a de-briefing session, re-thinking the processes undertaken provide uses meta-cognition reinforcement. In exploring the notion of average what questions did we ask that helped us to go deeper and deeper? Can we come up with a list of questions for a poster based on the four resource model headings of *Decoding, Meaning-making, Using and Analysing,* which we are going to use as we investigate other survey findings?

"It is really easy to think you are doing critical thinking because you are having an engaging discussion with your students. It is important to recognise that discussion can centre around 'meaning-making' and not get as far as 'analysing' or 'using'. As teachers we have to be deliberate in the questions we ask to encourage students to go deeper.

The temptation is to create scaffolded worksheets or activities for our students which we think will build up their skills, but without encouraging meta-cognition they may not have processes which they can transfer to new contexts. The biggest difficulty I encountered was being able to unpack my own thinking and realising what critical questions I found useful."

7. Independent practice Students can then use their *four resource critical numeracy model poster* as a lens in thinking about different news articles featuring average or other mathematical concepts. What do they see, think, wonder? What insights are they making about the issues with surveys? (A new Critical Numeracy website with lessons based around "Numeracy in the News" can be found at www.mercurynie.com.au/)

8. Authentic Survey/Study Using an issue current to the school, students can now work out how to get information that will help with decision making. How might this be integrated with another curriculum area? What additional skills might students need? (Examples include sampling, survey question design, graphing and graphical analysis techniques.) Students could then present findings to the school's decision making body, with section on limitations of the study, inviting feedback.